http://en.wikipedia.org/wiki/Fourier_transform

Operations

Operation	$x(t)$	$X(\omega)$
Scalar multiplication	$kx(t)$	$kX(\omega)$
Addition	$x_1(t) + x_2(t)$	$X_1(\omega) + X_2(\omega)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Scaling (a real)	$x(at)$	$\frac{1}{\lvert a\rvert} X\left(\frac{\omega}{a}\right)$
Time shifting	$x(t - t_0)$	$X(\omega)e^{-j\omega t_0}$
Frequency shifting (ω_0 real)	$x(t)e^{j\omega_0 t}$	$X(\omega - \omega_0)$
Time convolution	$x_1(t) * x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi} X_1(\omega) * X_2(\omega)$
Time differentiation	$\frac{d^n x}{dt^n}$	$(j\omega)^n X(\omega)$
Time integration	$\int_{-\infty}^{t} x(u)\,du$	$\frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$

Functional relationships

	Function	Fourier transform unitary, ordinary frequency	Fourier transform unitary, angular frequency	Fourier transform non-unitary, angular frequency	Remarks
	$f(x)$	$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i x\xi}\, dx$	$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\omega x}\, dx$	$\hat{f}(\nu) = \int_{-\infty}^{\infty} f(x) e^{-i\nu x}\, dx$	Definition
101	$a\cdot f(x) + b\cdot g(x)$	$a\cdot \hat{f}(\xi) + b\cdot \hat{g}(\xi)$	$a\cdot \hat{f}(\omega) + b\cdot \hat{g}(\omega)$	$a\cdot \hat{f}(\nu) + b\cdot \hat{g}(\nu)$	Linearity
102	$f(x-a)$	$e^{-2\pi i a\xi} \hat{f}(\xi)$	$e^{-ia\omega} \hat{f}(\omega)$	$e^{-ia\nu} \hat{f}(\nu)$	Shift in time domain
103	$e^{2\pi i a x} f(x)$	$\hat{f}(\xi - a)$	$\hat{f}(\omega - 2\pi a)$	$\hat{f}(\nu - 2\pi a)$	Shift in frequency domain, dual of 102
104	$f(ax)$	$\frac{1}{\|a\|} \hat{f}\left(\frac{\xi}{a}\right)$	$\frac{1}{\|a\|} \hat{f}\left(\frac{\omega}{a}\right)$	$\frac{1}{\|a\|} \hat{f}\left(\frac{\nu}{a}\right)$	Scaling in the time domain. If $\|a\|$ is large, then $f(ax)$ is concentrated around 0 and $\frac{1}{\|a\|} \hat{f}\left(\frac{\omega}{a}\right)$ spreads out and flattens.
105	$\hat{f}(x)$	$f(-\xi)$	$f(-\omega)$	$2\pi f(-\nu)$	Duality. Here $\hat{f}$ needs to be calculated using the same method as Fourier transform column. Results from swapping "dummy" variables of *x* and ξ or ω or μ.
106	$\frac{d^n f(x)}{dx^n}$	$(2\pi i\xi)^n \hat{f}(\xi)$	$(i\omega)^n \hat{f}(\omega)$	$(i\nu)^n \hat{f}(\nu)$	
107	$x^n f(x)$	$\left(\frac{i}{2\pi}\right)^n \frac{d^n \hat{f}(\xi)}{d\xi^n}$	$i^n \frac{d^n \hat{f}(\omega)}{d\omega^n}$	$i^n \frac{d^n \hat{f}(\nu)}{d\nu^n}$	This is the dual of 106
108	$(f * g)(x)$	$\hat{f}(\xi)\hat{g}(\xi)$	$\sqrt{2\pi} \hat{f}(\omega)\hat{g}(\omega)$	$\hat{f}(\nu)\hat{g}(\nu)$	The notation $f * g$ denotes the convolution of *f* and *g* — this rule is the convolution theorem
109	$f(x)g(x)$	$(\hat{f} * \hat{g})(\xi)$	$\frac{(\hat{f} * \hat{g})(\omega)}{\sqrt{2\pi}}$	$\frac{1}{2\pi}(\hat{f} * \hat{g})(\nu)$	This is the dual of 108
110	For $f(x)$ purely real	$\hat{f}(-\xi) = \overline{\hat{f}(\xi)}$	$\hat{f}(-\omega) = \overline{\hat{f}(\omega)}$	$\hat{f}(-\nu) = \overline{\hat{f}(\nu)}$	Hermitian symmetry. $\overline{z}$ indicates the complex conjugate.
111	For $f(x)$ a purely real even function	$\hat{f}(\omega)$, $\hat{f}(\xi)$ and $\hat{f}(\nu)$ are purely real even functions.			
112	For $f(x)$ a purely real odd function	$\hat{f}(\omega)$, $\hat{f}(\xi)$ and $\hat{f}(\nu)$ are purely imaginary odd functions.			

Square-integrable functions

	Function	Fourier transform unitary, ordinary frequency	Fourier transform unitary, angular frequency	Fourier transform non-unitary, angular frequency	Remarks
	$f(x)$	$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x)e^{-2\pi ix\xi}\,dx$	$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{-i\omega x}\,dx$	$\hat{f}(\nu) = \int_{-\infty}^{\infty} f(x)e^{-i\nu x}\,dx$	
201	$\operatorname{rect}(ax)$	$\frac{1}{\|a\|}\cdot \operatorname{sinc}\left(\frac{\xi}{a}\right)$	$\frac{1}{\sqrt{2\pi a^2}}\cdot \operatorname{sinc}\left(\frac{\omega}{2\pi a}\right)$	$\frac{1}{\|a\|}\cdot \operatorname{sinc}\left(\frac{\nu}{2\pi a}\right)$	The rectangular pulse and the *normalized* sinc function, here defined as sinc(x) = sin(πx)/(πx)
202	$\operatorname{sinc}(ax)$	$\frac{1}{\|a\|}\cdot \operatorname{rect}\left(\frac{\xi}{a}\right)$	$\frac{1}{\sqrt{2\pi a^2}}\cdot \operatorname{rect}\left(\frac{\omega}{2\pi a}\right)$	$\frac{1}{\|a\|}\cdot \operatorname{rect}\left(\frac{\nu}{2\pi a}\right)$	Dual of rule 201. The rectangular function is an ideal low-pass filter, and the sinc function is the non-causal impulse response of such a filter.
203	$\operatorname{sinc}^2(ax)$	$\frac{1}{\|a\|}\cdot \operatorname{tri}\left(\frac{\xi}{a}\right)$	$\frac{1}{\sqrt{2\pi a^2}}\cdot \operatorname{tri}\left(\frac{\omega}{2\pi a}\right)$	$\frac{1}{\|a\|}\cdot \operatorname{tri}\left(\frac{\nu}{2\pi a}\right)$	The function tri(x) is the triangular function
204	$\operatorname{tri}(ax)$	$\frac{1}{\|a\|}\cdot \operatorname{sinc}^2\left(\frac{\xi}{a}\right)$	$\frac{1}{\sqrt{2\pi a^2}}\cdot \operatorname{sinc}^2\left(\frac{\omega}{2\pi a}\right)$	$\frac{1}{\|a\|}\cdot \operatorname{sinc}^2\left(\frac{\nu}{2\pi a}\right)$	Dual of rule 203.
205	$e^{-ax}u(x)$	$\frac{1}{a+2\pi i\xi}$	$\frac{1}{\sqrt{2\pi}(a+i\omega)}$	$\frac{1}{a+i\nu}$	The function $u(x)$ is the Heaviside unit step function and a>0.
206	$e^{-\alpha x^2}$	$\sqrt{\frac{\pi}{\alpha}}\cdot e^{-\frac{(\pi\xi)^2}{\alpha}}$	$\frac{1}{\sqrt{2\alpha}}\cdot e^{-\frac{\omega^2}{4\alpha}}$	$\sqrt{\frac{\pi}{\alpha}}\cdot e^{-\frac{\nu^2}{4\alpha}}$	This shows that, for the unitary Fourier transforms, the Gaussian function exp($-\alpha x^2$) is its own Fourier transform for some choice of α. For this to be integrable we must have Re(α)>0.
207	$\mathrm{e}^{-a\|x\|}$	$\frac{2a}{a^2+4\pi^2\xi^2}$	$\sqrt{\frac{2}{\pi}}\cdot\frac{a}{a^2+\omega^2}$	$\frac{2a}{a^2+\nu^2}$	For a>0. That is, the Fourier transform of a decaying exponential function is a Lorentzian function.
208	$\operatorname{sech}(ax)$	$\frac{\pi}{a}\operatorname{sech}\left(\frac{\pi^2}{a}\xi\right)$	$\frac{1}{a}\sqrt{\frac{\pi}{2}}\operatorname{sech}\left(\frac{\pi}{2a}\omega\right)$	$\frac{\pi}{a}\operatorname{sech}\left(\frac{\pi}{2a}\nu\right)$	Hyperbolic secant is its own Fourier transform
209	$e^{-\frac{a^2x^2}{2}}H_n(ax)$	$\frac{\sqrt{2\pi}(-i)^n}{a}\cdot e^{-\frac{2\pi^2\xi^2}{a^2}}H_n\left(\frac{2\pi\xi}{a}\right)$	$\frac{(-i)^n}{a}\cdot e^{-\frac{\omega^2}{2a^2}}H_n\left(\frac{\omega}{a}\right)$	$\frac{(-i)^n\sqrt{2\pi}}{a}\cdot e^{-\frac{\nu^2}{2a^2}}H_n\left(\frac{\nu}{a}\right)$	H_n is the Hermite's polynomial. If a = 1 then the Gauss-Hermite functions are eigenfunctions of the Fourier transform operator. For a derivation, see Hermite polynomial. The formula reduces to 206 for n = 0.

Distributions

	Function	Fourier transform unitary, ordinary frequency	Fourier transform unitary, angular frequency	Fourier transform non-unitary, angular frequency	Remarks
	$f(x)$	$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x)e^{-2\pi i x\xi}\,dx$	$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{-i\omega x}\,dx$	$\hat{f}(\nu) = \int_{-\infty}^{\infty} f(x)e^{-i\nu x}\,dx$	
301	1	$\delta(\xi)$	$\sqrt{2\pi}\cdot\delta(\omega)$	$2\pi\delta(\nu)$	The distribution $\delta(\xi)$ denotes the Dirac delta function.
302	$\delta(x)$	1	$\frac{1}{\sqrt{2\pi}}$	1	Dual of rule 301.
303	e^{iax}	$\delta\left(\xi - \frac{a}{2\pi}\right)$	$\sqrt{2\pi}\cdot\delta(\omega - a)$	$2\pi\delta(\nu - a)$	This follows from 103 and 301.
304	$\cos(ax)$	$\frac{\delta\left(\xi - \frac{a}{2\pi}\right) + \delta\left(\xi + \frac{a}{2\pi}\right)}{2}$	$\sqrt{2\pi}\cdot\frac{\delta(\omega - a) + \delta(\omega + a)}{2}$	$\pi\left(\delta(\nu - a) + \delta(\nu + a)\right)$	This follows from rules 101 and 303 using Euler's formula: $\cos(ax) = (e^{iax} + e^{-iax})/2.$
305	$\sin(ax)$	$\frac{\delta\left(\xi - \frac{a}{2\pi}\right) - \delta\left(\xi + \frac{a}{2\pi}\right)}{2i}$	$\sqrt{2\pi}\cdot\frac{\delta(\omega - a) - \delta(\omega + a)}{2i}$	$-i\pi\left(\delta(\nu - a) - \delta(\nu + a)\right)$	This follows from 101 and 303 using $\sin(ax) = (e^{iax} - e^{-iax})/(2i).$
306	$\cos(ax^2)$	$\sqrt{\frac{\pi}{a}}\cos\left(\frac{\pi^2\xi^2}{a} - \frac{\pi}{4}\right)$	$\frac{1}{\sqrt{2a}}\cos\left(\frac{\omega^2}{4a} - \frac{\pi}{4}\right)$	$\sqrt{\frac{\pi}{a}}\cos\left(\frac{\nu^2}{4a} - \frac{\pi}{4}\right)$	
307	$\sin(ax^2)$	$-\sqrt{\frac{\pi}{a}}\sin\left(\frac{\pi^2\xi^2}{a} - \frac{\pi}{4}\right)$	$\frac{-1}{\sqrt{2a}}\sin\left(\frac{\omega^2}{4a} - \frac{\pi}{4}\right)$	$-\sqrt{\frac{\pi}{a}}\sin\left(\frac{\nu^2}{4a} - \frac{\pi}{4}\right)$	
308	x^n	$\left(\frac{i}{2\pi}\right)^n \delta^{(n)}(\xi)$	$i^n\sqrt{2\pi}\delta^{(n)}(\omega)$	$2\pi i^n\delta^{(n)}(\nu)$	Here, *n* is a natural number and $\delta^{(n)}(\xi)$ is the *n*-th distribution derivative of the Dirac delta function. This rule follows from rules 107 and 301. Combining this rule with 101, we can transform all polynomials.
309	$\frac{1}{x}$	$-i\pi\,\operatorname{sgn}(\xi)$	$-i\sqrt{\frac{\pi}{2}}\operatorname{sgn}(\omega)$	$-i\pi\,\operatorname{sgn}(\nu)$	Here sgn(ξ) is the sign function. Note that 1/*x* is not a distribution. It is necessary to use the Cauchy principal value when testing against Schwartz functions. This rule is useful in studying the Hilbert transform.
310	$\frac{1}{x^n} := \frac{(-1)^{n-1}}{(n-1)!}\frac{d^n}{dx^n}\log\lvert x\rvert$	$-i\pi\frac{(-2\pi i\xi)^{n-1}}{(n-1)!}\operatorname{sgn}(\xi)$	$-i\sqrt{\frac{\pi}{2}}\cdot\frac{(-i\omega)^{n-1}}{(n-1)!}\operatorname{sgn}(\omega)$	$-i\pi\frac{(-i\nu)^{n-1}}{(n-1)!}\operatorname{sgn}(\nu)$	$1/x^n$ is the homogeneous distribution defined by the distributional derivative $\frac{(-1)^{n-1}}{(n-1)!}\frac{d^n}{dx^n}\log\lvert x\rvert$

311	$\lvert x\rvert^{\alpha}$	$-2\frac{\sin(\pi\alpha/2)\Gamma(\alpha+1)}{\lvert 2\pi\xi\rvert^{\alpha+1}}$	$\frac{-2}{\sqrt{2\pi}}\frac{\sin(\pi\alpha/2)\Gamma(\alpha+1)}{\lvert\omega\rvert^{\alpha+1}}$	$-2\frac{\sin(\pi\alpha/2)\Gamma(\alpha+1)}{\lvert\nu\rvert^{\alpha+1}}$	This formula is valid for 0 > α > −1. For α > 0 some singular terms arise at the origin that can be found by differentiating 318. If Re α > −1, then $\lvert x\rvert^{\alpha}$ is a locally integrable function, and so a tempered distribution. The function $\alpha \mapsto \lvert x\rvert^{\alpha}$ is a holomorphic function from the right half-plane to the space of tempered distributions. It admits a unique meromorphic extension to a tempered distribution, also denoted $\lvert x\rvert^{\alpha}$ for α ≠ −2, −4, ... (See homogeneous distribution.)
312	$\operatorname{sgn}(x)$	$\frac{1}{i\pi\xi}$	$\sqrt{\frac{2}{\pi}}\frac{1}{i\omega}$	$\frac{2}{i\nu}$	The dual of rule 309. This time the Fourier transforms need to be considered as Cauchy principal value.
313	$u(x)$	$\frac{1}{2}\left(\frac{1}{i\pi\xi}+\delta(\xi)\right)$	$\sqrt{\frac{\pi}{2}}\left(\frac{1}{i\pi\omega}+\delta(\omega)\right)$	$\pi\left(\frac{1}{i\pi\nu}+\delta(\nu)\right)$	The function $u(x)$ is the Heaviside unit step function; this follows from rules 101, 301, and 312.
314	$\sum_{n=-\infty}^{\infty}\delta(x-nT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(\xi-\frac{k}{T}\right)$	$\frac{\sqrt{2\pi}}{T}\sum_{k=-\infty}^{\infty}\delta\left(\omega-\frac{2\pi k}{T}\right)$	$\frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta\left(\nu-\frac{2\pi k}{T}\right)$	This function is known as the Dirac comb function. This result can be derived from 302 and 102, together with the fact that $\sum_{n=-\infty}^{\infty}e^{inx} = 2\pi\sum_{k=-\infty}^{\infty}\delta(x+2\pi k)$ as distributions.
315	$J_0(x)$	$\frac{2\,\operatorname{rect}(\pi\xi)}{\sqrt{1-4\pi^2\xi^2}}$	$\sqrt{\frac{2}{\pi}}\cdot\frac{\operatorname{rect}\left(\frac{\omega}{2}\right)}{\sqrt{1-\omega^2}}$	$\frac{2\,\operatorname{rect}\left(\frac{\nu}{2}\right)}{\sqrt{1-\nu^2}}$	The function $J_0(x)$ is the zeroth order Bessel function of first kind.
316	$J_n(x)$	$\frac{2(-i)^nT_n(2\pi\xi)\operatorname{rect}(\pi\xi)}{\sqrt{1-4\pi^2\xi^2}}$	$\sqrt{\frac{2}{\pi}}\frac{(-i)^nT_n(\omega)\operatorname{rect}\left(\frac{\omega}{2}\right)}{\sqrt{1-\omega^2}}$	$\frac{2(-i)^nT_n(\nu)\operatorname{rect}\left(\frac{\nu}{2}\right)}{\sqrt{1-\nu^2}}$	This is a generalization of 315. The function $J_n(x)$ is the n-th order Bessel function of first kind. The function $T_n(x)$ is the Chebyshev polynomial of the first kind.
317	$\log\lvert x\rvert$	$-\frac{1}{2}\frac{1}{\lvert\xi\rvert}-\gamma\delta(\xi)$	$-\frac{\sqrt{\pi/2}}{\lvert\omega\rvert}-\sqrt{2\pi}\gamma\delta(\omega)$	$-\frac{\pi}{\lvert\nu\rvert}-2\pi\gamma\delta(\nu)$	γ is the Euler–Mascheroni constant.
318	$(\mp ix)^{-\alpha}$	$\frac{(2\pi)^{\alpha}}{\Gamma(\alpha)}u(\pm\xi)(\pm\xi)^{\alpha-1}$	$\frac{\sqrt{2\pi}}{\Gamma(\alpha)}u(\pm\omega)(\pm\omega)^{\alpha-1}$	$\frac{2\pi}{\Gamma(\alpha)}u(\pm\nu)(\pm\nu)^{\alpha-1}$	This formula is valid for 1 > α > 0. Use differentiation to derive formula for higher exponents. u is the Heaviside function.

Two-dimensional function

	Function	Fourier transform unitary, ordinary frequency	Fourier transform unitary, angular frequency	Fourier transform non-unitary, angular frequency
400	$f(x,y)$	$\hat{f}(\xi_x,\xi_y) = \iint f(x,y)e^{-2\pi i(\xi_x x+\xi_y y)}\,dx dy$	$\hat{f}(\omega_x,\omega_y) = \frac{1}{2\pi}\iint f(x,y)e^{-i(\omega_x x+\omega_y y)}\,dx dy$	$\hat{f}(\nu_x,\nu_y) = \iint f(x,y)e^{-i(\nu_x x+\nu_y y)}\,dx dy$
401	$e^{-\pi\left(a^2x^2+b^2y^2\right)}$	$\frac{1}{\|ab\|}e^{-\pi\left(\xi_x^2/a^2+\xi_y^2/b^2\right)}$	$\frac{1}{2\pi\cdot\|ab\|}e^{\frac{-\left(\omega_x^2/a^2+\omega_y^2/b^2\right)}{4\pi}}$	$\frac{1}{\|ab\|}e^{\frac{-\left(\nu_x^2/a^2+\nu_y^2/b^2\right)}{4\pi}}$
402	$\mathrm{circ}(\sqrt{x^2+y^2})$	$\frac{J_1\left(2\pi\sqrt{\xi_x^2+\xi_y^2}\right)}{\sqrt{\xi_x^2+\xi_y^2}}$	$\frac{J_1\left(\sqrt{\omega_x^2+\omega_y^2}\right)}{\sqrt{\omega_x^2+\omega_y^2}}$	$\frac{2\pi J_1\left(\sqrt{\nu_x^2+\nu_y^2}\right)}{\sqrt{\nu_x^2+\nu_y^2}}$

Formulas for general n-dimensional functions

	Function	Fourier transform unitary, ordinary frequency	Fourier transform unitary, angular frequency	Fourier transform non-unitary, angular frequency
500	$f(x)$	$\hat{f}(\xi) = \int_{\mathbf{R}^n} f(x)e^{-2\pi i x\cdot\xi}\,d^n x$	$\hat{f}(\omega) = \frac{1}{(2\pi)^{(n/2)}}\int_{\mathbf{R}^n} f(x)e^{-i\omega\cdot x}\,d^n x$	$\hat{f}(\nu) = \int_{\mathbf{R}^n} f(x)e^{-ix\cdot\nu}\,d^n x$
501	$\chi_{[0,1]}(\|x\|)(1-\|x\|^2)^\delta$	$\pi^{-\delta}\Gamma(\delta+1)\|\xi\|^{-(n/2)-\delta}$ $\cdot J_{n/2+\delta}(2\pi\|\xi\|)$	$2^{-\delta}\Gamma(\delta+1)\,\|\omega\|^{-(n/2)-\delta}$ $\cdot J_{n/2+\delta}(\|\omega\|)$	$\pi^{-\delta}\Gamma(\delta+1)\left\|\frac{\nu}{2\pi}\right\|^{-(n/2)-\delta}$ $\cdot J_{n/2+\delta}(\|\nu\|)$
502	$\|x\|^{-\alpha},\quad 0<\mathrm{Re}\ \alpha<n.$	$c_\alpha\|\xi\|^{-(n-\alpha)}$		